Linear Transceiver Design for MIMO Relay Broadcast Channels with Max-Min Fairness

*Edwin Monroy and †Sunghyun Choi
*International IT Policy Program, College of Engineering, Seoul National University, Seoul, Korea
†Department of Electrical and Computer Engineering and INMC, Seoul National University, Seoul, Korea
Email: emonroy@mwnl.snu.ac.kr, schoi@snu.ac.kr

Abstract—The multiple-input multiple-output relay broadcast channel with one source, one relay, and two destinations is studied. It is assumed that the source and the relay cooperatively serve one of two destinations, which may have a weaker channel from the source (due to, for example, being located far from the source), while the source alone serves the other destination that has a stronger channel. Given the asymmetry in the use of the relay as well as in the channel strengths, the performance of the system is evaluated through the minimum of the rates for the two destinations instead of the sum of the rates. A new linear transmit and receive beamforming scheme is proposed for this max-min fairness problem that can be implemented in practical scenarios. Numerical results show that the proposed scheme yields a significant gain over existing schemes.

Index Terms—MIMO relay broadcast channel, half-duplex relay, max-min fairness, interference alignment, repetition coding.

I. INTRODUCTION

The relay broadcast channel (RBC) is a variation of the classic three-node relay channel [1] in which a transmitter sends messages to a number of receivers with the help of one or more relays. Starting from the information-theoretic work in [2], several papers study the RBC and propose diverse transmit and receive beamforming schemes for practical half-duplex relays with multiple-input multiple-output (MIMO) nodes, such as [3], [4], among many other. These studies, however, neglect the direct link between the sources and destinations, so that the source serves all the destinations via the relay only. The authors of [5] consider a scenario where users are divided into direct and relay users that are served directly by the source and through the relay, respectively. Here the direct link between source and relay users is also omitted. Other papers that do consider the direct link such as [6] assume that the relay and source transmits data to two different groups of users separately to avoid interference at the expense of the rate of the system. Moreover, as in most of the existing work, the relay operates in amplify-forward (AF) mode. In contrast, there are much fewer studies for the decode-and-forward (DF) RBC. In addition to [2], our previous work [7] and some references therein study several configurations of the RBC with a DF relay.

In general, the goal of all these existing results is to maximize sum rate. However, when power, spatial streams (SSs), or even the time slot duration is to be optimally allocated, and if the strength of the links to the users varies greatly, as happens in practice, the better users would be allotted most, if not all, of the resources at the expenses of the rest. The authors in [8] consider achieving fairness for a RBC by maximizing the minimum signal-to-noise ratio (SNR) of all users. A relaying technique based on user selection, a non-linear transceiver design, and power allocation is proposed. Again, an AF relay is considered and all the users are served by source and relay. Besides, the proposed scheme employs Tomlinson-Harashima precoding, which is a relatively complex non-linear technique, and requires an iterative solution. In addition, source and relay do not cooperate in serving the users.

In contrast, in this paper we consider an RBC with one source, two destinations, and a single DF relay, which is used to serve only one of the destinations (D1, which may be, e.g., located at the cell edge) cooperatively with the source. The other destination (D2) does not need help from the relay and thus receives its signals from the source only. This configuration models a scenario that could arise in practice, e.g., in the downlink of cellular systems. Our objective is to maximize the minimum rate between D1 and D2, i.e., to calculate the max-min rate of the system. We propose a new transmission strategy and formulate its max-min rate as an optimization problem. Due to the high complexity of the resulting problem, we put forward a non-iterative, linear transceiver design that fully eliminates all the interference and then allocate the available resources, namely, time-slot duration, available SSs at each link, and transmit power at source and relay to obtain the highest rate possible.

The contribution of this paper is two-fold: (1) we study an RBC scenario that may arise in the downlink of a single cell in cellular systems with the goal of achieving max-min rate fairness among users; and (2) we propose a transmission scheme and transceiver design with resource allocation based on practical/simplifying techniques such as linear beamforming, non-iterative algorithms, and repetition coding.

The rest of the paper is structured as follows. Our system model and its achievable max-min rate are introduced in Sections II and III whereas the proposed transceiver design and its resulting rate are presented in Section IV. Some numerical results are shown in Section V while our conclusion

This work was supported by the National Research Foundation of Korea (NRF) grant funded by the Korean government (MEST) No. 2013031071. The authors are also very grateful to Prof. Young-Han Kim at UC San Diego, who provided valuable insight and expertise that greatly improved this paper.
follows in Section VI.

II. SYSTEM MODEL

We consider a system with one source, one DF half-duplex relay, and two destinations, each one with \( M \) antennas.\(^1\) The MIMO channels from source to relay (S-R), source to each destination (S-D1 and S-D2, also called direct links), and relay to each destination (R-D1 and R-D2) are referred to as \( H_{SR}, H_{SD1}, H_{SD2}, H_{RD1}, \) and \( H_{RD2} \) \( \in \mathbb{C}^{M \times M} \), respectively, and are assumed to be perfectly known at all nodes. These matrices can be further expressed as \( H_i = \sqrt{\gamma_i} H_i \), \( i = \{SR, SD1, SD2, RD1, RD2\} \), where \( H_i \) is a i.i.d circular symmetric complex normal Gaussian random matrix, and \( \gamma_i = d_i^{-\alpha} \) is the path loss (\( d_i \) is the distance between the two nodes on link \( i \)), \( \alpha \), the path loss exponent). The source and relay nodes are subject to maximum transmit power constraints \( P_S \) and \( P_R \), respectively.

The proposed transmission protocol is illustrated in Fig. 1, which shows the transmitted signals before precoding. The relay operates in half-duplex mode in time and in the first time slot, the duration of which is denoted by \( t \), the source transmits signal \( x_S^{(1)} = F_F^{(1)} x_F^{(1)} + F_D1 x_D1^{(1)} + F_D2 x_D2^{(1)} \), where \( x_F^{(1)} \in \mathbb{C}^{m_{i1} \times 1} \), \( x_D1^{(1)} \in \mathbb{C}^{m_{i1} \times 1} \), and \( x_D2^{(1)} \in \mathbb{C}^{m_{i2} \times 1} \) are independent, complex normal Gaussian vectors to be decoded by the relay and each destination, respectively; \( F_F^{(1)} \in \mathbb{C}^{M \times m_{i1}} \), \( F_D1^{(1)} \in \mathbb{C}^{M \times m_{i1}} \), and \( F_D2^{(1)} \in \mathbb{C}^{M \times m_{i2}} \) are precoding matrices; and \( m_{i1}, m_{i1}^{(1)}, \) and \( m_{i2}^{(1)} \) are the number of SSs allocated to each signal.

The received signals at relay, D1, and D2 in the first time slot are given, respectively, by

\[
\begin{align*}
x_R &= H_{SR} x_S^{(1)} + n_R^{(1)}, \\
x_{D1} &= H_{SD1} x_S^{(1)} + n_{D1}^{(1)}, \\
x_{D2} &= H_{SD2} x_S^{(1)} + n_{D2}^{(1)},
\end{align*}
\]

where \( n_R^{(1)} \) and \( n_{D1}^{(1)}, \) \( i = 1, 2, \) represent the normal Gaussian noise vectors at the relay and the destinations. In the second time slot (with a duration of \( 1-t \)), the relay obtains an estimate of the source message from \( x_R \), referred to as \( x_F^{(2)} \), which is then precoded and forwarded to D1 as \( x_D1^{(2)} = G_R x_F^{(2)} \), where \( G_R \in \mathbb{C}^{M \times m_{i}} \) is the relay precoding matrix, and \( m_{i} \) is the number of SSs of \( x_R \). In our system, source and relay cooperate in transmitting \( x_F^{(2)} \) to D1 (as in [10]); hence, the source also sends a precoded copy of \( x_F^{(2)} \) in the second time slot. In addition, the source transmits a new signal, \( x_{D1}^{(2)} \) to D1.

The two signals are sent over the eigenmodes of the S-D1 link, as shown in Fig. 1b. Finally, the source also transmits a new signal, \( x_{D2}^{(2)} \) to D2.

The source signal in the second time slot is then given by \( x_S^{(2)} = F_F^{(2)} x_F^{(2)} + F_D1 x_D1^{(2)} + F_D2 x_D2^{(2)} \), where \( F_F^{(2)} \in \mathbb{C}^{M \times m_{i1}^{(1)}}, F_D1^{(2)} \in \mathbb{C}^{M \times m_{i1}^{(1)}}, F_D1^{(2)} \in \mathbb{C}^{M \times m_{i2}^{(1)}}, \) and \( F_D2^{(2)} \in \mathbb{C}^{M \times m_{i2}^{(1)}} \) are the source precoding matrices in the second time slot; and \( x_F^{(2)} \in \mathbb{C}^{m_{i1}^{(1)} \times 1}, x_D1^{(2)} \in \mathbb{C}^{m_{i1}^{(1)} \times 1}, \) and \( x_D2^{(2)} \in \mathbb{C}^{m_{i2}^{(1)} \times 1} \) are independent, complex normal Gaussian vectors. Moreover, \( m_{i1}^{(1)}, m_{i1}^{(1)} \) and \( m_{i2}^{(1)} \), represent the number of SSs of each signal. Note that \( m_{i} = m_{i1}^{(1)} \), because relay and source transmit the same signal \( x_F^{(2)} \) to D1. The precoding matrices at source and relay shall meet the power constraints \( \text{tr}(F_F^{(2)} F_F^{(2)H}) + \text{tr}(F_D1^{(2)} F_D1^{(2)H}) + \text{tr}(F_D2^{(2)} F_D2^{(2)H}) \leq P_S, i = 1, 2, \) and \( \text{tr}(G_R G_R^{H}) \leq P_R \), respectively. Also, \( m_{i1}^{(1)} + m_{i1}^{(1)} + m_{i2}^{(1)} = M \).

Finally, the signals received at D1 and D2 in the second time slot are, respectively,

\[
\begin{align*}
y_{D1} &= H_{SD1} x_S^{(2)} + H_{RD1} x_R + n_{D1}^{(2)}, \\
y_{D2} &= H_{SD2} x_S^{(2)} + H_{RD2} x_R + n_{D2}^{(2)},
\end{align*}
\]

and \( n_{D1}^{(2)}, \) \( i = 1, 2, \) is the normal Gaussian noise vector at Di.

III. ACHIEVABLE MAX-MIN RATE

It can be shown that the achievable rate region of our system can be obtained from [2, Theorem 2] and [10], as the convex closure of the set of rates \( (R_1, R_2) \) to each destination D1 and D2 respectively, such that

\[
R_1 < R_1^{\max} = \min \left\{ t I(x_F^{(1)}; y_R) + (1-t) I(x_F^{(2)}; y_{D1}^{(2)} | x_R), \right. \\
\left. t I(x_F^{(1)}; y_{D1}^{(1)}) + (1-t) I(x_F^{(2)}; y_{D1}^{(2)}; y_{D2}^{(2)}) + t I(y_{D1}^{(1)}; y_{D1}^{(2)}), \right. \\
\left. (1-t) I(y_{D1}^{(1)}; y_{D2}^{(2)}), \right. \\
R_2 < R_2^{\max} = t I(x_F^{(2)}; y_{D2}^{(2)}) + (1-t) I(x_{D2}^{(2)}; y_{D2}^{(2)})
\]

where it is assumed that D1 decodes \( x_F^{(2)} \) first. We are interested in the max-min rate of the system, denoted by \( R_{\text{maxmin}} \), which is to be optimized over the time-slot duration, the source and relay precoding matrices, and the number of SSs allocated to the input signals:

\[
R_{\text{maxmin}} = \max_{\substack{t \in (0,1), F_F^{(1)}, F_D1^{(1)}, F_D2^{(1)}}} \min( R_1^{\max}, R_2^{\max}) \quad (3)
\]

subject to constraints on power and the number of SSs in both time slots. The expressions for the mutual information terms in \( R_1^{\max} \) and \( R_2^{\max} \) are omitted due to limited space.

We may attempt to carry out this highly non-convex optimization in its current form (3), but it seems intractable to obtain a global (or even a local) optimal solution. Therefore, in the next section we propose a simpler, suboptimal solution based on linear transceivers with resource allocation, and calculate the resulting max-min rate of the system.

IV. LINEAR TRANSCEIVER DESIGN

In this section, we design the transmit and receive beamformers separately for each time slot, and then we obtain the max-min rate of the system with the proposed beamformers.

\(^1\)This assumption is commonly made when schemes based on interference alignment (IA) are considered, such as [9].
A. First Time Slot

The transceiver design in the first time slot is based on the block-diagonalization (BD) technique with coordinated transmit-receive processing, similar to [7]. The relay, D1, and D2 decoders are set to $U_R = U_{SR}^{m_{d1}}$, $U_{D1} = U_{SD1}^{m_{d1}}$, and $U_{D2} = U_{SD2}^{m_{d2}}$, respectively, where $U_{SR}$, $U_{SD1}$, and $U_{SD2}$ are the matrices of left-singular vectors from the singular value decomposition (SVD) of the channel matrices of the S-R, S-D1, and S-D2 links, respectively. $A|b_a$ denotes columns from $a$ to $b$ of matrix $A$. Also, the source precoders are defined as

$$F_{j} = V_{j} \tilde{B}_{j} F_{j}^{1/2} \rightleftharpoons j = F, D1, \text{ and } D2$$

where the first term precancels the interfering signals at each receiver, whereas the other two are used to perform SVD and water-filling power allocation on the equivalent, interference-free channels (refer to [7] for the details).

As for power allocation, notice that if we were to maximize sum rate, the source power would be allocated via water filling with $\text{tr}(P_{F} + P_{D1} + P_{D2}) \leq P_S$. However, since the objective in this paper is to maximize the minimum rate of the system, such power allocation strategy may not be appropriate. Instead, we set $\text{tr}(P_{F} + P_{D1}) \leq F_{d1}$, and $\text{tr}(P_{D2}) \leq F_{d2}$, where $P_{f,1}^{(1)}$ and $P_{d}^{(1)}$ define the portion of $P_S$ allocated to $x_{F}^{(1)}$, $x_{D1}$, and $x_{D2}$, respectively, such that $P_{f,1}^{(1)} + P_{d}^{(1)} + P_{d2}^{(1)} \leq P_S$. These three parameters are chosen to optimize problem (3). After this, water filling is used to distribute each signal’s allocated power over their respective SSs.

B. Second Time Slot

In the second time slot, signals $x_{F}^{(2)}$ from the relay and source and $x_{D}^{(2)}$ from the source cause interference at D2, and so does $x_{D}^{(2)}$ at D1. In order to remove these unwanted signals, we employ an approach inspired by IA [7]. The proposed transceivers consist of two stages that are designed independently: the first stage performs coordinated spatial multiplexing (CSM) to eliminate the interference at the receivers [9]; the second stage performs single-user SVD and water filling over the resulting channels. Accordingly, we express the transmit precoders as

$$F_{F}^{(2)} = F_{F \text{IA}}^{(2)} F_{SF}^{(2)} F_{SU}^{(2)}$$

$$F_{D1}^{(2)} = F_{D1 \text{IA}}^{(2)} F_{D1SU}^{(2)}$$

$$F_{D2}^{(2)} = F_{D2 \text{IA}}^{(2)} F_{D2SU}^{(2)}$$

and the receive decoders as

$$U_{F}^{(2)} = U_{D1IA}^{(2)} U_{D1SU}^{(2)}$$

$$U_{D2}^{(2)} = U_{D2IA}^{(2)} U_{D2SU}^{(2)}.$$  

Notice that $U_{F}^{(2)}$ is of size $M \times m_{d1}^{(2)}$, where $m_{d1}^{(2)} = m_{d1}^{(1)} + m_{d1}^{(2)}$. We also define $F_{F \text{FIA}}^{(2)} = [F_{F}^{(1)} F_{D1}^{(1)}] F_{F \text{FIA}}^{(1)} F_{F \text{FIA}}^{(1)}$. Here, the CSM part of the transceivers is denoted by subindex IA and the single-user part is denoted by SU. In order to fully nullify interference and, at the same time, maximize the performance at each receiver, it can be shown that the first stage of the transceivers should satisfy the following CSM equations:

$$\mathcal{C} \left( F_{F \text{FIA}}^{(2)} \right) \perp \mathcal{C} \left( U_{D2IA}^{(2)} H_{SD2} \right),$$

$$\mathcal{C} \left( F_{D1IA}^{(2)} \right) \perp \mathcal{C} \left( U_{D1IA}^{(2)} H_{SD1} \right),$$

$$\mathcal{C} (G_{RIA}) \perp \mathcal{C} \left( U_{D2IA}^{(2)} H_{RD2} \right),$$

where $\mathcal{C}(A)$ denotes the column space of $A$. These equations can be solved to obtain $U_{D2IA}^{(2)} = \mathcal{O}([b_1, \ldots, b_{m_{d1}^{(2)}}])$, where $b_i$ is one of the $M$ eigenvectors of matrix $B = \left( H_{RD2} H_{SD2} \right)^{-1} H_{RD2} H_{SD1}$, and $\mathcal{O}(A)$ is an orthonormal base for $\mathcal{C}(A)$. Since any $m_{d1}^{(2)}$ eigenvectors of $B$ satisfy the CSM equations, we find the ones that, according to (3), maximize the minimum rate via a full search of size $(M)$. The remaining transceivers are computed from the same CSM equations.

We now focus on the second stage of the transceivers. By plugging the CSM stage of the transceivers into (1) and (2), the received signal at each destination now becomes

$$U_{D1SU}^{(2)} = U_{D1SU}^{(2)} \left( H_{SD1} F_{F \text{IA}}^{(2)} F_{F}^{(2)} + H_{RD1} G_{R} x_{F}^{(2)} \right) + \tilde{H}_{SD1} x_{D1}^{(2)} + \tilde{n}_{D1}^{(2)}$$

$$U_{D2SU}^{(2)} = U_{D2SU}^{(2)} \left( H_{SD2} F_{F \text{IA}}^{(2)} F_{F}^{(2)} + H_{RD2} G_{R} x_{F}^{(2)} \right) + \tilde{H}_{SD2} x_{D2}^{(2)} + \tilde{n}_{D2}^{(2)}$$

where $\tilde{n}_{D1} = U_{D1}^{(2)} n_{D1}^{(2)}$, and $\tilde{n}_{D2} = U_{D2}^{(2)} n_{D2}^{(2)}$. Also, $H_{SD1} = U_{D1IA}^{(2)} F_{F \text{IA}}^{(2)}$, $H_{RD1} = U_{D1IA}^{(2)} F_{F \text{IA}}^{(2)}$, $H_{SD2} = U_{D2IA}^{(2)} F_{F \text{IA}}^{(2)}$, and $H_{RD2} = U_{D2IA}^{(2)} F_{F \text{IA}}^{(2)}$. It can be noticed that after applying the CSM stage, both destinations see an equivalent channel with no interference. However, while at D2 it is a simple SU-MIMO channel over which only $x_{D2}^{(2)}$ is received, D1 sees two equivalent channels: one from source and relay, $H_{D1} = [H_{SD1} H_{RD1}]$, over which two copies of $x_{F}^{(2)}$ are received and have to be appropriately combined; and the other from the source, $H_{SD1}$, over which $x_{F}^{(2)}$ is conveyed.

Additionally, we follow the same strategy used in the first time slot regarding power allocation. We define $P_{f,2}^{(2)}$, $P_{d}^{(2)}$, and $P_{d2}^{(2)}$ as the fraction of $P_S$ allocated to $x_{F}^{(2)}$, $x_{D1}^{(2)}$, and $x_{D2}^{(2)}$, respectively, with $P_{f,2}^{(2)} + P_{d}^{(2)} + P_{d2}^{(2)} \leq P_S$. Again, the three parameters are chosen to optimize (3).

Then, for D2 we can apply SVD with water filling to obtain the single-user part of the respective transceivers as

$$U_{D2SU}^{(2)} = U_{SD2}^{(2)} F_{F \text{IA}}^{(2)} F_{F}^{(2)}$$

where $U_{SD2}$ and $V_{SD2}$ are the matrix of left and right-singular vectors of $H_{SD2}$ respectively, and $P_{d2}^{(2)}$ is a diagonal power allocation matrix obtained by water filling over the singular values of $H_{SD2}$, subject to $\text{tr}(P_{SD2}^{(2)}) \leq P_{d2}^{(2)}$.

As for D1, we follow an approach similar to that in [10]. We first set $F_{D1SU}^{(2)} = V_{SD1}^{(2)} F_{D1}^{(2)}$, where $V_{SD1}$ is the matrix of right-singular vectors of $H_{SD1}$ and $F_{D1}^{(2)}$ is obtained by water filling with $\text{tr}(P_{SD1}^{(2)}) \leq P_{d1}^{(2)}$. Now, we let D1 filter out $x_{D2}^{(2)}$ by applying whitening filter $K_{D1}^{(2)} x_{D2}^{(2)}$ (before applying the single-user decoder), where $K_{D1}^{(2)} = I_{m_{d2}^{(2)}} + H_{SD1} F_{F \text{IA}}^{(2)} F_{F}^{(2)} H_{SD1}^{H}$, $I_{m_{d2}^{(2)}}$ thus, only $x_{D1}^{(2)}$ remains and the equivalent composite channel from source and relay to D1 now
becomes $K_{D1,2}^{-\frac{1}{2}}H_{D1}$. If we initially assume that the relay and the source can pool their transmit power for $x_i^{(2)}$ as $P_f^{(2)} + P_R$, then the composite source-relay precoder can be calculated by SVD with water filling, as $F_{SU} = V_{D11}^{\dagger}P_{D1}^{\dagger}$, where $V_{D11}$ is the matrix of right-singular vectors of $K_{D1,2}^{-\frac{1}{2}}H_{D1}$, and $P_{D1}$ is the water-filling power matrix, with $tr(P_{D1}) \leq P_f^{(2)} + P_R$. Now, let $F_{SU}$ be a matrix containing the first $m_f^{(2)}$ rows of $F_{SU}$ and $G_{SU,2}$ the remaining $m_f^{(2)}$ rows. We can finally obtain the single-user matrices as $F_{SU}^{(2)} = \sqrt{\alpha_1}F_{SU,1}$, $G_{SU}^{(2)} = \sqrt{\alpha_2}G_{SU,2}$, where the factors $\alpha_1$ and $\alpha_2$ scale the composite source-relay precoding matrix to satisfy the individual power constraints of source and relay respectively, and are given by

$$\alpha_1 = \frac{P_f^{(2)}}{tr(F_{SU}^{(2)}F_{SU,1}^{\dagger})}, \quad \alpha_2 = \frac{P_R}{tr(G_{SU,2}G_{SU,2}^{\dagger})}.$$ 

Finally, $U_{D1,SU}^{(2)} = U_{D1}$, with $U_{D1}$ being the matrix of left-singular vectors of $K_{D1,2}^{-\frac{1}{2}}H_{D1}$.

C. Max-min Rate of the Proposed Scheme

We now determine the max-min rate that our scheme can achieve with the proposed transceivers from the original problem [3]. Since the decoders at the relay, D1, and D2 are unitary matrices, we can plug the expressions for the proposed transceivers in [3], and after rearranging some terms, obtain $R_{\text{max}}$, the max-min rate of our scheme:

$$R_{\text{max}} = \max_{i \in \{0,1\}, m_i^{(d)}, m_i^{(d)} \in \mathbb{Z}_{\geq 0}, \quad P_f^{(i)}, P_d^{(i)} \in \mathbb{R}_{\geq 0}, i = 1, 2} \min(R_{\text{max}}^{(1)}, R_{\text{max}}^{(2)})$$

where

$$R_{\text{max}}^{(1)} = \min\left((a_3 + a_1)t + a_2(1-t), a_1t + a_4(1-t)\right)$$

$$R_{\text{max}}^{(2)} = b_1t + b_2(1-t)$$

s.t. $m_i^{(d)} + m_{d1}^{(i)} + m_{d2}^{(i)} = M, P_f^{(i)} + P_d^{(i)} = P_S, i = 1, 2$

$$a_i = R \left(U_{D1}^{(i)\dagger}H_{SD1}, F_{D1}^{(i)}\right), b_i = R \left(U_{D2}^{(i)\dagger}H_{SD2}, F_{D2}^{(i)}\right), i = 1, 2$$

$$a_3 = R \left(U_{RS}^{(i)\dagger}F_{F}^{(1)}\right),$$

$$a_4 = R \left(U_{D1}^{(i)\dagger}H_{SD1}F_{D1}^{(i)}, U_{D1}^{(i)\dagger}H_{D1}Q^{(2)}\right)$$

and $H_{D1} = [H_{SD1}, H_{RD1}]$.

$$Q^{(2)} = \begin{bmatrix} F_F^{(2)} F_F^{(2)H} & F_F^{(2)} G_R^{H} \\ G_R F_F^{(2)H} & G_R G_R^{H} \end{bmatrix}$$

$$R(A, B) = \log_2 \left(\det(I + ABB^H A_1^H)\right)$$

$$R(A_1, B, A_2, C) = \log_2 \left(\det(I + A_1BB^H A_1^H + A_2CA_2^H)\right)$$

Solving this problem seems very challenging. Since the SS allocation variables $m_i^{(d)}$, $m_{d1}^{(i)}$, and $m_{d2}^{(i)}$ $i = 1, 2$ take on integer values, [5] is a mixed-integer non-linear problem, which is known to be very difficult to solve optimally [12]. In order to reduce complexity when M is small ($\leq 4$, for example), we perform an exhaustive search over all possible combinations of $m_i^{(d)}$, $m_{d1}^{(i)}$, and $m_{d2}^{(i)}$ and numerically solve the problem over $t, P_f^{(i)}, P_d^{(i)}$, and $P_S$. We set $M = 4, P_S = P_R = P$ normalized to the noise power, $n = 3$, and the results are averaged over 50 independent channel realizations. We compare the max-min rate of the proposed scheme obtained as described in Section [IV-C] (denoted by “Proposed” in the figures) with that of the following two schemes:

1) Direct link: The two destinations are served directly by the source, and the relay is not used. Such a setup corresponds to a MIMO BC channel with two receivers, and, hence, we use the same technique employed in Section [IV-A] with two receivers in this case.

2) Multihopping: The S-D1 link is not used and all transmissions from source to D1 are completed exclusively via the relay. In the first time slot we employ the same design in Section [IV-A] with the source transmitting to relay and D2 only. Similarly, the design in Section [IV-B] for the second time slot is reused and adapted to this case. This scheme is equivalent to that in [5] with two users, a DF relay, optimal power allocation and time slot duration, and MIMO channels.

We evaluate the three schemes under a commonly used scenario modified for our setup, where the S-D1 distance, $d_{SD1}$, is normalized to one and the relay is located along the line connecting the two nodes. Thus, $d_{RD1}$, the R-D1 distance, is equal to $1 - d_{SR}$, and we let $d_{SR}$, the S-R distance, be equal to $d = [0.1, 0.9]$. D2 is located along the same line, at a distance $d_{SD2}$ from the source.

In Figs. [2][3] we compare the max-min rates of the three schemes when D2 is located close ($d_{SD2} = 0.2$) or far ($d_{SD2} = 1$) from the source and when the transmit power is either low ($P = 0$) or relatively high ($P = 10$). The figures
show that the proposed scheme outperforms the other two, especially when the relay is close to the source and the transmit power is low. For instance, in Fig. 2, the rate of the proposed scheme is about 70% higher than that of the direct link when $d = 0.1$. However, when $P = 10$ dB, Fig. 3 shows that the gain is reduced to around 30%. This can be explained by the fact that, when the relay is close to the source, the rate of the system is dominated by what happens in the second time slot. In this slot, the proposed scheme yields higher diversity gain at D1 than the other schemes by having relay and source cooperatively transmit signal $x_2^{(2)}$. Clearly, the effect of this diversity gain is more pronounced in the low-SNR regime.

We can also see that the max-min rate of multihopping is higher than that of direct link at low power for given $d_{SD2}$. When power is low, using the relay to reach D1 yields a better rate than direct transmission, whereas when power increases, the loss introduced by the half duplex operation of the relay is larger than the gain it can achieve. However, for given $P$, as D2 moves away from the source, the rate of multihopping becomes higher than that of direct link, as Fig. 4 shows. The reason is that with increasing $d_{SD2}$ and fixed $P$, the SNR on the S-D2 link decreases and drives the max-min rate down; hence, using the relay for D1 can yield a higher rate for D1 than direct transmission and result in a higher max-min rate.

Finally, we can observe in Fig. 5 that when the objective function in (5) is the sum rate (“Sum rate maximization”), the minimum rate of the system, that of D1 in this case, is extremely low (similarly low rates are obtained in the scenarios in Figs. 5 and 6 and are omitted for clarity). As mentioned before, this is due to the asymmetry in the quality of the links between the source and the two destinations, so that when sum rate is maximized, most resources are allocated to the better destination, resulting in an extremely unfair allocation to D1. In contrast, by achieving max-min fairness, both destinations are always able to achieve a non-trivial rate.

VI. CONCLUSION

In this paper, we have studied a practical configuration of the RBC that models the downlink of a single cell in a cellular system with a base station, a relay, and two types of users, e.g., cell-edge and non-cell-edge users. We have proposed a new transmission strategy and formulated an optimization problem to obtain its max-min rate, unlike previous work that aims to obtain the maximum sum rate. Given the complexity of the max-min problem, we have proposed a suboptimal solution with lower complexity based on a non-iterative, linear precoding/decoding scheme that relies on BD and IA to completely remove the interference with resource allocation. The numerical results show that our system outperforms other schemes by up to 70%, especially in the low-SNR regime and when the relay is located close to the source.

REFERENCES