Throughput and Energy Consumption Analysis of IEEE 802.15.4 Slotted CSMA/CA

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We propose a new analytic model of the IEEE 802.15.4 slotted CSMA/CA from which throughput and energy consumption are computed in saturation conditions. The analytic results are validated via ns-2 simulations.

Introduction: IEEE 802.15.4 is designed for low data-rate and small-size wireless personal area networks (WPANs) [1]. This letter focuses on the star topology of 802.15.4 expected to be a dominant type at least in the foreseeable future. In the topology, each device uses a slotted carrier sense multiple access (CSMA/CA). In view of using binary exponential backoff, the CSMA/CA is similar to the IEEE 802.11 CSMA/CA. However, in 802.15.4, a backoff counter value of the device decreases regardless of the channel status, and the device senses the channel, i.e., performs Clear Channel Assessment (CCA), two times when the value reaches zero. The throughput and energy consumption are major performance metrics for 802.15.4. In [2], a Markov chain model of the 802.15.4 is proposed, where each state is based on the counter values as the 802.11 model in [3]. Both models describe the behavior of the protocols using the probability that the device is in the channel accessing states. However, in the 802.15.4, the probability is not suitable to describe the behavior because the channel sensing should be performed twice before entering the accessing states. Moreover, in the case of energy consumption, to the authors’ best knowledge, there is no known analysis.
In this letter, we propose a new Markov chain model of the 802.15.4, and analyze the throughput and energy consumption in saturation conditions. The proposed model utilizes the probability of a device in the channel sensing states instead of the channel accessing states.

**Analytical Model:** To analyze the 802.15.4 in saturation conditions, we propose a discrete time Markov chain model as shown in Fig. 1 under the same assumptions as in [2]. For a given end-device, we define $b(t)$ and $s(t)$, which are the stochastic processes representing a backoff counter and a backoff stage, respectively. We let $b_{i,j} = \lim_{t \to \infty} P\{s(t) = i, b(t) = j\}$, $i \in (0,m)$, $j \in (-1,W_i-1)$ where $m$ is the maximum stage, and $W_i = 2^{BE-\min(\beta E_{\text{max}} - \beta E_{\text{min}})}$. Now, $b_{i,0}$ and $b_{i,-1}$ are the states corresponding to the first CCA and the second CCA slots, respectively. The key approximation is that the busy probability of the channel at the first CCA and at the second CCA are $\alpha$ and $\beta$, respectively, regardless of the stages. Then, transition probabilities are obtained as follows:

$$
\begin{align*}
P\{b_{i,j} | b_{i,j+1}\} &= 1, \quad i \in (0,m), \quad j \in (0,W_i - 2) \\
P\{b_{i,-1} | b_{i,0}\} &= 1 - \alpha, \quad i \in (0,m), \\
P\{b_{i,j} | b_{i-1,j}\} &= \alpha / W_i, \quad i \in (1,m), \quad j \in (0,W_i - 1) \\
P\{b_{i,j} | b_{i-1,j-1}\} &= \beta / W_i, \quad i \in (1,m), \quad j \in (0,W_i - 1) \\
P\{b_{0,j} | b_{i,-1}\} &= (1-\beta) / W_0, \quad i \in (0,m-1), \quad j \in (0,W_0 - 1) \\
P\{b_{0,j} | b_{m,0}\} &= \alpha / W_0, \quad j \in (0,W_0 - 1) \\
P\{b_{0,j} | b_{m-1,0}\} &= 1 / W_0, \quad j \in (0,W_0 - 1)
\end{align*}
$$

(1)
To transmit a packet, when the given device performs CCAs, all devices should be in the backoff states \((b_{i,j}, j \in (0, W_i - 1))\). If the channel is idle for these two CCAs, the transmission states simply follow. Therefore, we do not model the transmission states, but focus on the probability that the device is in the sensing states. We define \(\tau\), the conditional probability that a device is at one of the first CCA states, \((b_{i,0})\), out of the backoff states as follows:

\[
\tau = \frac{\sum_{i=0}^{m} b_{i,0}}{\sum_{i=0}^{m} \sum_{j=0}^{W_i-1} b_{i,j}} = \frac{2(1-(\alpha + \beta - \alpha \beta)^{m+1})}{\sum_{i=0}^{m} (W_i^{\min_i} \cdot \max_i - \max_i \cdot \min_i) + 1(\alpha + \beta - \alpha \beta)}. \tag{2}
\]

This means that the device performs the first CCA with \(\tau\) if it is in one of the backoff states. When all \(n\) devices are in the backoff states, the probability \(\gamma\) that the given device transmits a packet successfully after performing CCAs twice is \((1-\tau)^{n-1}\). Then, the saturation throughput is derived as:

\[
S = \frac{n\tau(1-\alpha)(1-\beta)\gamma l_p}{(1-\tau) + \tau \alpha + 2\tau(1-\alpha)\beta + \tau(1-\alpha)(1-\beta)[\gamma T_s + (1-\gamma)T_c]}. \tag{3}
\]

where \(l_p\) is the payload length in the number of slots. \(T_s\) and \(T_c\) are the number of occupied slots for successful transmission and collision, respectively, and given by:

\[
T_s = 2\left[ T_{CCA} + \lceil T_L \rceil + \lceil \delta \rceil + \lceil T_{Ack} \rceil \right], \quad T_c = 2\left[ T_{CCA} + \lceil T_L \rceil \right] \tag{4}
\]

where \(T_{CCA}\), \(T_L\), \(\delta\) and \(T_{Ack}\) are time durations (in the number of slots) for performing a CCA, for transmitting \(L\)-slot packet, for waiting for an Ack and for receiving an Ack, respectively. Note that, in the 802.15.4, a device waits for an
Ack during $macAckWaitDuration$ (equal to 2.7 slots in 2.4GHz channel). However, we assume that the waiting duration is two slots after the last transmission slot. In addition, we also assume that the backoff procedure starts at the first ack waiting slot.

Now, $\tau$ depends on the state transition probabilities $\alpha$ and $\beta$. Since $\alpha$ is the busy probability at the first CCA, it is calculated using two average numbers: (1) the average number $T_o$ of backoff slots a device experiences to transmit a packet successfully, and (2) the average number $T_b$ of busy slots due to packet transmissions of other devices out of the backoff slots for $T_o$. $T_o$ and $T_b$ are derived from the relationship between the given device and others. When all devices are in the backoff states, the probability $P_{to}$ that the given device does not perform the first CCA and others perform the first CCA, and the probability $P_{so}$ that the transmission is successful are derived as:

$$P_{to} = (1 - \tau)[1 - (1 - \tau)^{n-1}], P_{so} = (n-1)\tau(1 - \tau)^{n-1} / P_{to} .$$

(5)

When the average number of backoff slots during the successful transmission and collision of others are $T_{os}$ and $T_{oc}$, respectively, $T_o$ is derived using (5) as:

$$T_o = \left[ P_{to} \{ P_{so} T_{os} + (1 - P_{so}) T_{oc} \} + \tau + (1 - P_{to} - \tau) \right] / \tau (1 - \tau)^{n-1} .$$

(6)

where $1/\tau(1 - \tau)^{n-1}$ is the average number of the first CCA attempts for the given device to transmit a packet successfully when all devices are in the backoff states. Since the device is not at the one of the backoff slots but at the
second CCA slots when the first CCA resulted in idle, if \( |\delta| = 1 \), \( T_{os} \) and \( T_{oc} \) are given by:

\[
T_{os} = T_s - (2\tau^2 + 2\tau(1 - \tau)), \quad T_{oc} = T_c - \tau
\]  

(7)

where \( 2\tau^2 \) represents the case of performing the first CCA in two cases: (1) when others perform the second CCA, and (2) when others wait for an Ack. Therefore, the given device performs the second CCA, and does not experience backoff for two slots. \( 2\tau(1 - \tau) \) represents the case of performing the first CCA at one of the above two cases. \( T_b \) is derived as follows:

\[
T_b = P_{ts} \left[ P_{so} T_{bs} + (1 - P_{so}) T_{bc} \right] / \tau (1 - \tau)^{a-1}
\]  

(8)

where \( T_{bs} \) and \( T_{bc} \) are the number of busy slots out of the backoff slots counted for \( T_{os} \) and \( T_{oc} \), and are given by \( T_{os} - 2 \left[ T_{CCA} \right] - \left[ \delta \right] \) and \( T_{oc} - 2 \left[ T_{CCA} \right] \), respectively. With (6) and (8), \( \alpha \) and \( \beta \) are derived as:

\[
\alpha = \frac{P_{ts} \left[ P_{so} T_{bs} + (1 - P_{so}) T_{bc} \right]}{P_{ts} \left[ P_{so} T_{os} + (1 - P_{so}) T_{oc} \right] + 1 - P_{ts}}
\]  

(9)

\[
\beta = \frac{P_{ts} \left[ P_{so} T_{os} + (1 - P_{so}) T_{oc} \right]}{P_{ts} \left[ P_{so} T_{os} + (1 - P_{so}) T_{oc} \right] + 1 - P_{ts}} \cdot \frac{1}{(1 - \alpha)}
\]  

(10)

where \( T_s = \left[ T_{CCA} \right] + \left[ \delta \right] \) and \( T_c = \left[ T_{CCA} \right] \).

In order to analyze energy consumption, we define the normalized energy consumption \( S_{Ed} \), which is the average energy consumption to transmit one-slot amount of payload. We use the parameter values in [4]. The energy
consumption $E_{Tx}$ to transmit, and $E_{Rx}$ to receive a packet during one slot are 0.0100224 mJ and 0.0113472 mJ, respectively. We set the energy consumption $E_{ta}$ during the turnaround time $T_{ta}$ as $(E_{Rx} + E_{Tx})/2$. We assume that a device enables its receiver only when it performs a CCA to transmit a packet or tries to fetch a pending packet from the coordinator. Except for those cases, zero energy consumption is assumed. Then, the consumed energy for the successful transmission $E_s$ and the collision $E_c$ are given by:

$$
E_s = 2T_{CCA}E_{Rx} + T_{ta}E_{ta} + T_{L}E_{Tx} + T_{wa}E_{wa} + (\delta - T_{wa} + T_{ack})E_{Rx}
$$

$$
E_c = 2T_{CCA}E_{Rx} + T_{ta}E_{ta} + T_{L}E_{Tx} + T_{wa}E_{wa} + (T_{L} - T_{wa} + 2)E_{Rx}
$$

When a device selects 1 as the backoff counter value after collision with probability $(1-\tau)$, it waits for an Ack in the receiving mode after a $TurnaroundTime$. In this case, the energy consumption for the first CCA is calculated twice. The terms $-(1-\tau)T_{CCA}E_{Rx}$ and $-\tau(2T_{CCA} + T_{wa})E_{Rx}$ in (11), are added to consider the cases of selecting the counter value 1 and 0, respectively. With (11), $S_{Ed}$ is derived as:

$$
S_{Ed} = \frac{\tau\alpha T_{CCA}E_{Rx} + 2\tau(1-\alpha)\beta T_{CCA}E_{Rx} + \tau(1-\alpha)(1-\beta)[\gamma E_s + (1-\gamma)E_c]}{\tau(1-\alpha)(1-\beta)\gamma l_p}.
$$

**Model Validation:** In order to validate the proposed model, we compare our results with ns-2 simulations'. We use default parameter values defined for 2.4GHz frequency channels such as 3, 5 and 4 for $macMinBE$, $aMaxBE$ and $macMaxCSMABackoff$, respectively. We consider a star topology with one coordinator and end devices, where the end devices transmit fixed size
packets to the coordinator. We also configure a long beacon interval (i.e., 15.4s) to focus on the CSMA/CA protocol itself. Fig. 2 shows the results when the packet lengths are 7 slots and 13 slots, respectively, and the overhead (header and footer) in each packet is 1.5 slots.

In all cases, the analytical results well match with the simulation results. To compare our model with the model in [3], we derive the saturation throughput in the same manner as (3) and (4). The results from [3] are notably different from the simulation results.

References

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Figure legends

Figure 1. Markov chain model of IEEE 802.15.4 slotted CSMA/CA

Figure 2. Saturation throughputs and energy consumptions

- $L=7$, throughput, simulation
- $L=7$, throughput, analysis
- $L=7$, energy, simulation
- $L=7$, energy, analysis
- $L=7$, Misic’s throughput, analysis

- $L=13$, throughput, simulation
- $L=13$, throughput, analysis
- $L=13$, energy, simulation
- $L=13$, energy, analysis
- $L=13$, Misic’s throughput, analysis
Fig. 1

Fig. 2